OTHER THEMES

Teacher Training: reflections of mathematical education in higher education

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ABSTRACT – Teacher Training: reflections of mathematical education in higher education. This paper maps the productions of the Mathematics Teachers Training generated by the Working Group on Mathematical Education in Higher Education, of the Brazilian Society of Mathematical Education. This investigation is aimed to analyze, among other aspects, the initial training of Mathematics teachers, the role of supervised internship practice and the development of the teaching professional. We sought to highlight the concerns that researchers, whose main field of interest is teaching in Higher Education, have about the initial and continued training of Mathematics teachers. We have identified, by means of Content Analysis, six pillars in which the topics presented in the body of the analysis are concentrated.
Keywords: **Teacher Training. Mathematics. Mathematical Education. Higher Education. Mapping of Researches.**

RESUMO – Formação de Professor: reflexões da educação matemática no ensino superior. Este artigo mapeia as produções sobre Formação de Professores de Matemática geradas pelo Grupo de Trabalho Educação Matemática no Ensino Superior, da Sociedade Brasileira de Educação Matemática, e tem por objetivo realizar análises, dentre outros aspectos, a respeito da formação inicial do professor de Matemática, do papel do estágio supervisionado obrigatório e do desenvolvimento profissional docente. Buscamos evidenciar as inquietações que pesquisadores cuja área principal de interesse é o ensino em nível superior têm sobre a formação inicial e continuada de professores de Matemática. Identificamos, por meio de uma Análise de Conteúdo, seis eixos nos quais se aglutinam as temáticas apresentadas no corpus de análise.
Palavras-chave: **Formação de Professores. Matemática. Educação Matemática. Ensino Superior. Mapeamento de Pesquisas.**

Introduction

As teachers of Mathematics, mathematical educators and members of the *Grupo de Trabalho Educação Matemática no Ensino Superior* – GT04 (Working Group on Mathematical Education in Higher Education – WG04) of the *Sociedade Brasileira de Educação Matemática* – SBEM (Brazilian Society of Mathematical Education), created in 2000, we felt the need to map the production of this WG since its origin. We have covered works presented by its members in the six editions of the *Seminário Internacional de Pesquisa em Educação Matemática* – SIPEM (International Seminar of Research in Mathematical Education), as well as two books and a thematic number of a scientific journal organized by the Group. This mapping, still under construction, is part of a wider project, which covered an initial stage constituted by the identification, in a *corpus* of 139 articles, of the authors of these works, of the institutions to which they are affiliated, of the topics, of the mathematical object dealt with and of the research subjects involved. In this initial survey, we identified eight investigations on the *Training of Mathematics Teachers*, which will be thoroughly analyzed in this article.

Among SBEM's Working Groups, there is one that specifically focuses on *Training of Teachers who Teach Mathematics* – WG07. It is important to notice what issues are emphasized about this topic in a WG that does not explicitly aim at this subject, namely, what are the concerns of researchers whose main field of interest is teaching in Higher Education regarding the initial and continuous training of Mathematics teachers. Besides being relevant for Mathematical Education, inventories like the one proposed by us can contribute to establish a closer dialogue between WG04 and WG07 and also to the perception of common features, divergences and specificities of the researches produced in both Groups due to their main interests. Mapping researches enables, among other aspects, to comprehend and systematize what already has been investigated in a certain field and to know how the academic production historically evolves. It is also possible to establish relations between the different researches already held and consequently to identify recurrent topics and to point out new perspectives. In the methodological point of view, we carried out a research based on Content Analysis, following Bardin's (2001) conception, detailed ahead.

Methodology

Content Analysis (Bardin, 2001) was used to collect, categorize and analyze data regarding the productions of WG04 on the Training of Mathematical Teachers. This methodology includes the following procedures: pre-analysis, exploration of the material and treatment and interpretation of the obtained results. The first of them, according to Bardin (2001, p. 121), "[...] is the organizational stage itself", which "[...] goal is to operationalize and systematize the initial thoughts, in a way that leads to a precise layout of the development of the successive ope-

rations, in a scoping plan". This stage is formed by actions such as: *(i)* the choice of the materials to be used, *(ii)* the formulation of hypotheses and objectives and *(iii)* the elaborations of indicators that found the final interpretation.

In this research's case, we decided beforehand that the *corpus*, defined by Bardin (2001, p. 122) as "[...] the set of documents taken into account to be submitted to analytical procedures", would be formed by the full texts of the works about the Training of Mathematical Teachers presented in WG04, in the editions of SIPEM, by the articles regarding this subject in the books *Educação Matemática no Ensino Superior: pesquisas e debates* (Mathematical Education in Higher Education: researches and debates – (Frota; Nasser, 2009)) and *Marcas da Educação Matemática no Ensino Superior* (Marks of Mathematical Education in Higher Education – (Frota; Bianchini, 2013; Carvalho, 2013)) Also, by the texts about the topic in volume 15, number 3, published in 2013, of the journal *Educação Matemática Pesquisa* – EMP (Mathematical Education Research), of the *Programa de Estudos Pós-Graduados em Educação* Matemática (Mathematical Post-Graduate Study Program) of PUC-SP.

About the rules highlighted by Bardin (2001) for the selection of materials to be analyzed, we can claim that the *corpus* of analysis considered in this work is *representative*, as the selected sample includes all the major publications of WG04 about the topic at hand. It is also *homogeneous*, as none of the documents presents specificities regarding the choice criteria, and *pertinent*, as it is a source of collecting data, the documents are appropriate to the objectives of investigation held.

Table 1 shows general information about the analyzed texts. We entered in contact with the author of the first work listed in Table 1 to access the complete text, but as we did not receive an answer, we limited ourselves to weave the considerations that were available to us, based on its abstract available on the website of SBEM.

Table 1 – General Information about the Analyzed Texts

Title	Authors	Publication and date
A representação social em alunos concluintes de curso de Licenciatura em Matemática de Instituições de Ensino Superior da Região Metropolitana do Recife sobre as características do professor de Matemática (The social representation in senior students of the undergraduate Mathematics program of Institutions of Higher Education from Recife about the characteristics of the Mathematics teacher)	Josinalva Estacio Menezes	I SIPEM, *Annals*, 2000
Representações sociais acerca da formação do bom professor de Matemática (Social representation about the training of the good Mathematics teacher)	Marger da Conceição Ventura Viana	IV SIPEM, *Annals*, 2009
Resolução de problemas na Licenciatura em Matemática – rumo à compreensão e à aquisição das grandes ideias contidas na Matemática Escolar (The resolution of problems in the undergraduate Mathematics program - towards the comprehension and the acquisition of the big ideas contained in School Mathematics)	Lourdes de la Rosa Onuchic, Norma Suely Gomes Allevato	IV SIPEM, *Annals*, 2009
Formação de professores – mudanças urgentes na Licenciatura em Matemática (Teacher training - urgent changes in the undergraduate Mathematics program)	Lourdes de la Rosa Onuchic, Norma Suely Gomes Allevato	Book: *Educação Matemática no Ensino Superior: pesquisas e debates, 2009*
Pesquisa sobre a própria prática no Ensino Superior de Matemática (Research about one's own practice in Higher Education Mathematics)	Gilda de La Rocque Palis	Book: *Educação Matemática no Ensino Superior: pesquisas e debates, 2009*
A (Trans)Formação pelo Estágio Supervisionado Obrigatório em um Curso de Licenciatura em Matemática (The (Trans)Formation through Supervised Internship Practice in an undergraduate Mathematics program)	Ana Márcia Fernandes Tucci de Carvalho	*Educação Matemática Pesquisa*, v. 15, n. 3, 2013
Resolução de problemas na formação inicial de professores de Matemática (The resolution of problems in the initial training of Mathematics teachers)	Lourdes de la Rosa Onuchic, Rosilda dos Santos Morais	*Educação Matemática Pesquisa*, v. 15, n. 3, 2013
A Licenciatura em Matemática: o desenvolvimento profissional dos formadores de professores (The Mathematics Degree: the professional development of the educators of teachers)	Lourdes de la Rosa Onuchic, Roger Huanca	Book: *Marcas da Educação Matemática no ensino superior, 2013*

Source: Drafted by the authors.

Once the texts were selected, we made a *fluctuating reading*, which according to Bardin (2001, p. 122) "[...] consists in establishing contact with the documents to be analyzed and in knowing the text, allowing oneself to be invaded by impressions and orientations". Without establishing a hypothesis beforehand, we undertook this reading keeping in consideration the goal of highlight the issues that permeate the considerations about the *Training of Mathematical Teachers* held by WG04 members. From this reading, *indexes* emerged and, according to Bardin (2001, p. 126) they can be explicit references to determined topics, which guided us in "[...] operations of *cutouts of the text* in comparable units of *categorization* for thematic analysis".

Next, we began to approach the materials that form the *corpus* of our research, or in the words of Bardin (2001), to *codify* it, an action that includes the cutting out of the text in the so-called *analysis units* and the definition of the categories of analysis, based on the classification and the aggregation of these cutouts. In our case, the cutouts happened by *register units*, which are "[...] units of signification to be codified and correspond to segments of content to be considered as base units" (Bardin, 2001, p. 130). As register units we have specifically adopted the different topics addressed regarding the Training of Mathematics Teachers in the analyzed texts.

These topics are related to aspects such as:

- What type of teacher would we like to form through a Mathematics degree?
- How should a course be structured to handle this training?
- What should the training of Mathematics teachers be and what concerns should they manifest in their work?
- What is the role of supervised internship practice in the training of the student?
- How can the graduated teachers professionally develop themselves?

These aspects refer specifically to the training of K-12 Education teachers, but in the analyzed texts, there is another topic regarding the Higher Education teaching of Mathematics practiced not necessarily in Mathematics degree programs. In this case, the importance of the Higher Education professor developing their researches in their own practices is highlighted.

Ultimately we proceeded to the *categorization* of data obtained from the selected texts. In other words, we gathered them in categories that, according to Bardin (2001), are categories that assemble groups of register units under a generic title due to the common characteristics of these elements. Thus, we defined the following thematic categories:

- C1: The good Mathematics teacher.
- C2: The initial training of the Mathematics teacher.
- C3: The supervised compulsory internship practice.

- C4: The training and the concerns manifested in the work of an educator of Mathematics teachers.
- C5: The professional development of the teacher.
- C6: Researches about one's own practice in Higher Education.

Below, we present considerations regarding each of the categories built.

Analyses Regarding the Categories of Analysis

Based on the reading of the selected works and guiding ourselves by the precepts of Content Analysis according to Bardin (2001), we defined six categories of analysis, comprising all aspects considered basic by us, regarding the topic *Training of Mathematics Teachers*. In this section, we present the analyses of the collected data.

C1: The good Mathematics Teacher

A first topic present in the analyzed texts regards the characteristics that should be present in those to which the authors refer to by different expressions, such as *good Mathematics teacher*, *competent teacher*, *proficient teacher* or *efficient teacher*.

By means of interviews with K-12 Education, High School and Higher Education teachers and with students of graduate and undergraduate Mathematics programs, Viana (2009) listed the necessary characteristics for a good teacher: the mastery of content (the most highlighted characteristic), the commitment with the task of educating, flexibility, autonomy, good humor (in the sense of expressing happiness with their job) and knowing the students' characteristics.

Onuchic and Allevato (2009b) highlight the consistent reflection of the teacher about their own work, about the profession and about the challenges found within it as essential characteristics of a good teacher. Based on Ponte's ideas (2002), they question: what does it mean being a good teacher and what knowledge and capacities do they need in an affective point of view and in a cognitive and social point of view? For the authors, it seems that it is more difficult to be a teacher nowadays when comparing to the past, as presently teachers "[...] have to deal with expertise, with technology, with the social complexity generated, partly due to the democratization of education and partly due to the fact that the society itself has difficulty in understanding what the role of the school is" (Onuchic; Allevato, 2009b, p. 19).

On the other hand, as Onuchic and Morais (2013, p. 674) highlight based on Silver's considerations (2006), nowadays there is "[...] a better idea of what being a 'qualified' Mathematics teacher means". For Onuchic and Allevato (2009a, p. 175), these professionals need:

> 1. A solid foundation in Statistics and Probability;
> 2. A solid foundation in plane, spatial and analytical Geometry, Geometry of transformation and Euclidean and non-Euclidean Geometry;

> 3. A solid basis in Calculus of a variable;
> 4. A good basis in Algebra and its functions (curve adjustments);
> 5. A certain familiarity with Discrete Mathematics (combinatorial analysis, recursive relationships, Graph theory, etc.);
> 6. Experience with diverse pedagogical perspectives;
> 7. Familiarity with technology. They should know how to use technology in meaningful ways. Technology is used in these curricula to facilitate the observation of models and relations and also in simulations.

The quote highlights that content knowledge is not enough, which is a consensus in Mathematical Education, even though it is a primordial prerequisite to those who teach Mathematics. Teachers "[...] should know well what they teach and should be able to justify what they do" (Onuchic; Huanca, 2013, p. 310). As Nóvoa (2001) emphasizes in a quote reproduced by Onuchic and Allevato (2009b), more than being an expert on the subject, it is necessary that the teacher is able to reorganize it, rework it and then carry out its transposition to a didactic situation in the classroom, adjusting it "[...] to the specific school activities of the different stages and modalities of K-12 Education" (Nóvoa, 2001 *apud* Onuchic, Allevato, 2009b, p. 8). These ideas are ratified by Onuchic and Morais (2013) who, based on Silver (2006), define a proficient Mathematics teacher as someone who:

> Has a deep knowledge in Mathematics, both outside and inside the school curriculum; knows how students learn and mentally operate mathematic knowledge [...], [has] a fluent repertoire in pedagogical procedures (procedure fluency to teach), together with the ability to plan classes and didactic materials and to evaluate in what way decisions and specific pedagogical actions can influence the learning of the students (strategic competency and adaptive reasoning to teach) (Onuchic; Morais, 2013, p. 674).

Menezes (2000) stated that, in undergraduate senior students' view, a good Mathematics teacher should, besides having domain of the content, search for a harmonious relationship with the students and paying attention to them, taking their individualities into consideration.

Regarding the desired characteristics for a Mathematics teacher, Onuchic and Allevato (2009b) also highlight contributions of Van de Walle (2001), to whom

> [...] truly effective teachers must include in their work four basic components: the appreciation of the Mathematics subject itself - which means "doing Mathematics"; the comprehension of how students learn and build ideas; the ability to plan and select tasks in a way that the students learn Mathematics in an environment of solving problems; and the ability of integrating evaluation into the process of teaching to increase and enhance learning in the day-to-day life (Onuchic; Allevato, 2009b, p. 9).

Onuchic and Huanca (2013, p. 329), when referring to the teacher that works with the *Methodology of Teaching-Learning-Evaluation of Mathematics through the Resolution of Problems*, list expected actions of a good teacher, with which we agree. Among others, "[...] to prepare or choose problems that are suitable with the content or the concept which is intended to be built, letting go of being the center of the activities, and passing to the students the responsibility through the learning they plan to reach."

These considerations highlight the need of reflecting upon how to form this *good*, *competent*, *proficient* or *efficient* teacher. Based on the analyzed researches, we will discuss next some aspects related to the initial training of a Mathematics teacher.

C2: The Initial Training of the Mathematics Teacher

Reading the documents that form the *corpus* of this research, it was evidenced, as expected, a higher frequency of investigations directed at reflections about the initial training of the Mathematics teacher in different aspects of the undergraduate Mathematics programs. It is discussed in these sources:

- The goals of these programs, taking into the consideration the need of differentiating the training of Mathematics teachers and the training of Mathematicians;
- The academic curricula (with emphasis on the role of Didactic of Mathematics and Resolution of Problems);
- The teacher's knowledge to be developed in the initial training;
- The continuation of the teacher's training after concluding their undergraduate program.

Reflecting upon the undergraduate Mathematics program, an initial aspect to consider is the profile of the professional who is graduating, thus a point to be observed is that "[...] a program of initial training of Mathematics teachers should necessarily be different from an undergraduate Mathematics program that aims at graduating mathematicians who dedicate mainly to investigation" (Onuchic; Allevato, 2009b, p. 6-7). Based on Ponte's ideas (2002), these authors emphasize that the initial training of teachers should aim at training professionals effectively competent for teaching, reflecting upon ways to improve the development of their professional practice.

Viana (2009) highlights a few goals to be considered in the Mathematics teacher training. Among them, providing for the future teacher the domain of content, psychological-pedagogical and/or didactic training, development of creativity and/or good humor, understanding of the relationship between theory and practice and opportunity to develop autonomy. Besides, it should be sought the training of teachers and researchers who educate. The academic curricula to be planned for Mathematic undergraduate programs should include space for the stu-

dents, as Onuchic and Huanca (2013, p. 313-314) emphasize, "[...] to live experiences of overcoming their mistaken conceptions in their initial education, in order to build solid mathematical knowledge and be able to diagnose them and help future students in this process of overcoming".

According to Viana (2009, p. 24) there is a need "[...] of making the academic curricula suitable to a type of training that conduces future Mathematics professionals to a wider comprehension of the learning and teaching processes". It is important to plan curricula that allow the student to realize that working with Mathematics in K-12 Education is more than developing abilities that only a reduced part will eventually use. It is basic to emphasize the role of Mathematics in society and its true nature and extension (Onuchic; Allevato, 2009a).

Based on the ideas of Pires (2002, p. 45), Onuchic and Allevato (2009b) recover three pillars, which should guide the curricular planning of an undergraduate Mathematics program. These pillars refer to the following aspects: "[...] (1) the concept of competence is central in the orientation of the initial training program for teachers; (2) it is indispensable that there is coherence in the training offered and the practice expected from the future teacher; (3) research is an essential element in the training of the teacher".

Regarding the first pillar, Onuchic and Allevato (2009b, p. 7) emphasize, among the considerations of Pires (2002), the premise that "[...] the formulation of a teacher's education program cannot have as a starting point a set of courses defined beforehand, [...] but should establish the professional competencies, which are expected that the teacher in training builds during their trajectory of training". Breaking the curricular logic in undergraduate programs is an idea defended also by Tardif (2010 *apud* Onuchic and Huanca, 2013), highlighting that, even when the organization of courses still persists at some moments, these should effectively be used to contribute to the education of the future teacher.

For instance, courses such as Calculus, Geometry or Algebra cannot be offered in Mathematics undergraduate programs without establishing relations between their contents and the ones which the students will work in classrooms in K-12 Education and High School (Onuchic; Huanca, 2013). For that matter, Onuchic and Morais (2013, p. 672) postulate that, if there is a reformulation in the way contents of teacher training is worked, "[...] what they learned at the Higher Education could eventually incorporate itself to their practice, as it had a direct relation with what they will teach".

The curricular paradigm can be broken, even regarding didactic issues that, due to their importance to the field, should not be restricted to a singular course, which is many times held at the end of the undergraduate program, as Onuchic and Allevato (2009b) highlight.

As for the second pillar proposed by Pires (2002), referring to the coherence between the education offered and the practice that is expected of the future teacher, Onuchic and Allevato (2009b, p. 9) draw at-

tention to a central aspect of teacher training: future teachers "[...] learn the profession in a similar place to where which they will act, although in an inverted situation". Consequently, recognizing the importance that the teacher of K-12 Education fully comprehends different strategies of teaching and learning, "[...] it is necessary to offer the students the opportunity of experiencing and thus, incorporating alternative methodologies of teaching and learning to their practice" (Onuchic; Allevato, 2009a, p. 184).

According to investigations held by Onuchic and Allevato (2009b, p. 19), "[...] active teachers quite often do not fully understand a teaching methodology that can present itself as favorable to the learning in the classroom". Aiming at minimizing this barrier, Onuchic and Morais (2013, p. 675) advocate that the experience of different methodological approaches in the development of activities proposed in undergraduate programs will offer to future teachers the opportunity to "[...] examine mathematical content with depth, reflecting about them in a way to aid the students' difficulties better and more frequently".

In the articles produced by WG04 that were analyzed, it is remarkable the predominance of discussions emphasizing the potentialities of the initial training of teachers and, consequently, of teaching in classrooms of K-12 Education through Resolution of Problems. Regarding the third pillar, Onuchic and Allevato (2009b) point out the need that teachers in initial training appropriate the results of researches in the field of Mathematical Education so these can, in the future, affect their actions in the classroom.

In addition to the goals of undergraduate programs and issues related to their academic curriculum, another aspect discussed by researchers of WG04 regarding the training of teacher who teach Mathematics refers to the teacher knowledge to be built in the initial training of these professionals. First, the full comprehension of mathematical contents (content knowledge) should evidently be sought, "[...] their meanings in different contexts and their interdisciplinary articulation (Onuchic; Allevato, 2009b, p. 7). But that alone is not enough!

Teaching with quality in K-12 Education is also the consequence of the construction of didactic, pedagogical knowledge, etc. by future teachers (Carvalho, 2013). Onuchic and Allevato (2009b, p. 5-6) synthesize this idea:

> The basic knowledge 'that lies behind' the teaching of Mathematics, the connections made between mathematical ideas, the students, the way students learn, the school culture where the work is being held, besides other pertinent factors in the school context, should necessarily include Mathematical knowledge.

The training of teachers who teach Mathematics does not finish once graduated, in the point of view of the authors in the analyzed articles. And that is due to several reasons.

Firstly, it happens due to issues directly related to fragilities in the current education offered in undergraduate programs. Onuchic and Allevato (2009b, p. 6-7) highlight that both Higher Education professors of specific areas (for instance, Mathematics) and teachers in the field of Education "[...] recognize that young teachers are not fully prepared in the courses they will teach", regarding the content or the strategies of teaching and learning. In the same way, the authors highlight, that in the active teachers' point of view, young teachers "[...] do not arrive sufficiently prepared in what they consider would be the most important". Onuchic and Morais (2013, p. 672) also point out that:

> The University has been rather disappointing in initial teacher training [and] the price paid is the unpreparedness of these teachers. In their majority, when put in the classroom to do the role to which they were assigned, they repeat old practices learned when they were students in K-12 Education, [...] as they were not offered the necessary assurance for the teaching practice in the years they were in the University (Onuchic; Morais, 2013, p. 672-673).

On the other hand, no matter how good the undergraduate Mathematics program offered to the future teachers is, it will still be impossible to claim that at its end the education will be complete.

As Onuchic and Morais (2013, p. 672) state, based on the contributions of Curi (2011), the teaching knowledge is "[...] derived from various sources and built in different moments". So, even though the Higher Education evidently has a fundamental role in the construction of professional knowledge for future teachers, there is knowledge to be built from the teaching practice and from continued education.

For Onuchic and Allevato (2009b, p. 6-7), in the point of view of teachers who are beginning their teaching practice, quite often learning through practice exceeds what was obtained in the initial training: "[...] new teachers regret that nothing that they learned in initial training helped them and that only in professional practice they learn what is important". Onuchic and Morais (2013, p. 672) also highlight that "[...] *the school* – where the teacher should begin their work as an educator to a great mass of the population – has been the place where they effectively learned to practice teaching".

Another reason pointed out by the authors for the initial training being, as its name indicates, only a first step in the teacher's training is that in a society, which constantly changes, such as the 21st century, "[...] only those who expand their professional education will excel" (Viana, 2009, p. 1). An inherent fundamental activity in undergraduate programs and a topic necessary to reflect upon is the supervised internship practice, which will be discussed ahead.

C3: The Supervised Internship Practice

This topic is discussed in only one of the analyzed works, the work of Carvalho (2013). Based on Pimenta (1997) and Fazenda's (1991, 2011)

ideas, the author highlights that the supervised internship practice, when the students can gather the theoretical academic knowledge built during their education with issues related to the school's everyday life, should be appreciated by Higher Education as well as the teaching institution that shall receive them.

> Specifically in the undergraduate Mathematics program, the internship allows to experience the multiplicity of issues involving teaching practice. We emphasize a few: experiencing the complexity of daily situations by means of observation; overcoming the difficulties of the requirements of preparing written reports from these observations; conceiving, organizing and selecting activities that are instigating workshops to provide non-standard Mathematical content regarding teaching and learning; finding the difficulties and surprises of classroom situations and of the interaction with the students to reflect upon the unique dynamic of this direction; transforming their own reflections, analyses and considerations in official reports (Carvalho, 2013, p. 632).

For the author, the supervised practice, "[...] hybrid space of opportunities of education, permeated by the educative intentionality" (Carvalho, 2013, p. 634), is the articulating pillar between the set of issues mentioned in the quote above, in which experiencing is the foundation of development of the future Mathematics teacher. The official documents also emphasize the importance of the supervised internship practice as one of the moments of pedagogical planning and of constitution of the social role of the future teacher (Carvalho, 2013).

In the critical training of the intern, Carvalho (2013, p. 635) emphasizes the relevance of observing the teacher's action in the classroom. This requires the student

> [...] to think about the attitudes of the students, about the attitudes of the teachers, about the planning held, about the physical structures of the teaching establishments. Observing the other to learn and comprehend, by means of observation, the dynamics of the Mathematics class.

Ultimately, the author advocated the rupture of the theory-practice dichotomy, by means of the activities in the supervised internship practice seeking a dialectical relation between these dimensions: "[...] theorizing about the practice as much as practicing the theory, without abandoning one in detriment to the other, without privileging one in detriment to the other, seeking in the overlapping of these areas the reflexive education necessary for the undergraduate student" (Carvalho, 2013, p. 643).

By discussing the initial training of the future teacher, looking at those who are responsible for that type of training is a necessary action and that has been evidenced in the analyzed texts. The next category highlights aspects about this topic.

C4: The Training and the Concerns Manifested in the Work of an Educator of Mathematics Teachers

One of the issues present in the discussions of WG04 related to the training of Mathematics Teacher regards what professionals should be responsible for teaching in the undergraduate Mathematics programs, what training should they have and what content approach concerns should they manifest in their classes. For Onuchic and Huanca (2013, p. 320), this issue deserves attention, as teachers who act in these programs work with a differentiated population – in the authors' words – which is formed by future teachers, being necessary to offer them "[...] a differentiated type of Mathematical education". For the authors, this could be the case if in the undergraduate program mainly "[...] mathematical educators with a good mathematical education would be present" (Onuchic; Huanca, 2013, p. 321). In other words, professionals with a master's or doctorate degree in Mathematical Education and, which in the courses under their responsibility, could call attention "[...] to the great mathematical ideas, [...] those responsible for the comprehension and for the meaning of different concepts, contents and constant operational techniques in the topics worked with in K-12 Education and High School" (Onuchic; Huanca, 2013, p. 320).

Onuchic and Huanca (2013) highlight the importance that the educator of teachers establish connections between the mathematical topics approached in Higher Education, whenever possible, and those, which the graduates will have to work with in K-12 Education. After all, there are different situations whereby the teacher can enable the students, "[...] in initial training, to realize the importance of an elementary knowledge in an advanced point of view" (Onuchic; Huanca, 2013, p. 328). For instance, when working with Algebra, the educator could analyze the different ways of conceiving the Mathematical field at hand, highlighting it as a study of binary relations about the set of objects, which according to the authors would allow future professionals to understand algebraic structures and to relate them to the Algebra taught in K-12 Education.

Likewise, "[...] in the Differential and Integral Calculus course, the important concept of limit is responsible for the understanding and for the meaning of various topics of K-12 Education" (Onuchic; Huanca, 2013, p. 321), as, for instance, understanding that 0,9999...=1. In other words, if in the teacher's initial training the concept of limit were explored taking into consideration that they are working with future teachers of K-12 Education or high school, then maybe they could realize that "[...] this is the concept responsible for justifying what is done only with rules when their operational techniques are worked with" (Onuchic; Huanca, 2013, p. 321-322).

Besides seeking to relate Higher Education content with those which the students will ultimately work with in K-12 Education, it is important, according to Onuchic and Allevato (2009b), to have space for the educators to be able to diagnose mistaken conceptions that the

students might have brought whilst entering the Higher Education and which can be considered as obstacles, and then helping them overcoming these barriers.

As Onuchic and Allevato (2009a, p. 183-184) point out, in fact, it is necessary for researchers to pay more attention to mathematical courses present in the undergraduate program.

> In the groups where Higher Education is discussed, most of the works presented aim at situations of Differential and Integral Calculus, even though they refer to the different treatment modalities due to the different ways of teaching in the many different undergraduate programs. A few works on Algebra and Geometry are also seen, but rarely about the training of teacher or the work in the classroom. In other words, the courses and mathematical contents that form the current undergraduate courses have not been investigated in the context of undergraduate programs, that is to say, under the perspective of teacher's training. [...] When alternative methodologies for practice in the classroom is mentioned, there is an impression that that is not very important to those who teach Mathematics in undergraduate courses. It is necessary to modify the way of facing changes in treating Mathematical content in the programs adopted, and the way they are being worked in the undergraduate programs themselves.

As we have stressed in the analyses of category C2, the teacher's training is not complete when completing the undergraduate program. The professional development of teachers happen along their career, as will be discussed below.

C5: The Professional Development of the Teacher

As Onuchic and Allevato (2009a, p. 174) highlight, based on the ideas of Oliveira (2003), the issue of the professional development of the teacher has been central in a range of works in the field of Teacher Training, "[...] advocating a perspective that puts the teachers in an active role in their education, contributing to effective changes in school mathematics" (Oliveira, 2003 *apud* Onuchic; Allevato, 2009a, p. 174) is also discussed by researchers of WG04.

Based on the ideas of Tardif (2010), Onuchic and Huanca (2013) highlight that with the goal of restructuring the main epistemological principles of the teaching profession, different paths have been walked. For instance, there have been reflections about the need of preparing a repertoire of knowledge for teaching based on the professional knowledge of teachers.

> This task suggests that University professors work with schools and in the classrooms in collaboration with the teachers, as co-researchers in the construction of their own professional knowledge. It is not always easy for the teacher to theorize their practice and to formalize their

> knowledge. For the researchers, the legitimization of the teacher's knowledge is far from having finished (Onuchic; Huanca, 2013, p. 319).

When thinking about professional development, according to Onuchic and Allevato (2009a, p. 174), based on Ponte (1998), other actions that are necessary are broadening and reviewing the conception of training, which "[...] should be useful to enable a diversity of paths and processes of professional development". For that matter, Onuchic and Huanca (2013, p. 319), based on Tardif (2010), highlight the importance of "[...] introducing mechanisms of training, action and research [...] suitable to the teachers and useful for their professional practice".

Clarke (1994) organized ten guiding principles for the planning and implementation of professional teaching developmental programs. These principles are described in the following way by Onuchic and Allevato (2009a, p. 172-173):

> 1. To list widely identified issues about concerns and interests on the subject (professional teaching development), even if not exclusively by the teachers themselves, and involving them, giving them a certain degree of choice;
> 2. To involve groups of teachers, more than individuals, from various schools, and recruiting school and regional administrative staff, students and parents and from the school community in general;
> 3. To recognize and discuss the many obstacles that prevent the improvement of teachers on a regional, school and individual level;
> 4. To consider teachers as participants in activities inside the classroom and students in real situations, shaping desired approaches in the classroom during in-service sessions, to project a clearer view of the changes suggested;
> 5. To request the conscious commitment of teachers for an active participation in the sessions of professional development, thus making them interested in the required readings and tasks appropriately adapted to their own classrooms;
> 6. To recognize what changes in the beliefs of teachers about teaching and learning are strongly derived from the practice in the classroom. As a result of these changes, they will continue the opportunity in validating, through the positive observation of the learning of students, information delivered by the professional developmental programs;
> 7. To dedicate time to plan, reflect upon and provide feedback, as to report accomplishments and failures to the group, sharing the knowledge of practice and discussing issues and solutions, observing the students individually and new teaching approaches;
> 8. To build capacity of the participating teachers and achieve a substantial level of comprehension, through the involvement in decision-making and by being seen as true partners in the process of change;

9. To recognize that change is a gradual, difficult and many times painful process, and enable opportunities for group support and for possible critiques;
10. To encourage the participants to establish future goals for their professional growth.

The professional teaching development is a continuing process in the career of K-12 Education teachers and Higher Education professors. Regarding this last topic, one of the analyzed articles highlights the development of researches about one's own practice as relevant for this process, which will be discussed next.

C6: Researches about One's Own Practice in Higher Education

In the analysis of the productions of WG04 about Mathematics Teacher training, aside from detecting issues related to the characteristics of a good Mathematics teacher, to the initial training of the Mathematics teacher, to the supervised internship practice, to the concerns to be manifested by an educator of Mathematics teacher in his work and in professional developmental teaching, we also observed that only two analyzed papers bring reflections upon the importance of Higher Education Mathematics professors developing researches about their own practice, in any undergraduate program.

It is important to observe that while in the other categories of analysis the reflections are directly aimed at the undergraduate Mathematics program, and consequently at teachers who will teach this course in K-12 Education, this sixth category includes wider aspects, related to teaching at Higher Education in any undergraduate program in which Mathematics is present. In our opinion, this is the category that truly reveals a specific approach to WG04 for the topic Training of Mathematics Teachers. It highlights that the researchers of the Groups reflect upon the topic in a wider way, without restricting themselves to the training of K-12 Education and High School teachers.

Based on the considerations of Cross (1986) and Cross and Steadman (1996), Palis (2009) states that in the education of the 21st century it is essential that Higher Education is aware of its responsibility for the quality of the teaching being offered to the students, which effectively has implications on their learning. For that matter, it is essential "[...] to discuss and stimulate the teacher's research about their own practice in Mathematic courses in Higher Education" (Palis, 2009, p. 203). The importance of teachers' research about their own practice is also emphasized by Onuchic and Huanca (2013), following Tardif's considerations (2010).

Palis (2009) also emphasizes that, way beyond sticking to recurrent sentences in the teacher's discourse to designate some of the difficulties faced by Higher Education students – such as *the student lacks a foundation*, *the student is weak* or *there are epistemological and pedagogical causes* – it becomes necessary that Mathematics departments pay more actual attention to these difficulties. "It is necessary to stimulate

the teachers [...] to look with a new light to their pedagogical efforts, embracing the idea, in Higher Education, of a teacher who researches his own practice" (Palis, 2009, p. 208). Regarding the work of this type of teacher, the author considers that they:

> [...] align investigation and teaching: when faced with a didactic problem, they submit it to a critical exam, solving it in the best possible way and announcing its solution. This work benefits the teachers themselves and the students, generating knowledge and developing the professional culture of reference communities (Palis, 2009, p. 204).

Based on Palis's considerations (2009), we finish the analyses of this category emphasizing that the researches about one's own practice can be made in the scope of each of the mathematical courses in Higher Education, as these present specificities in both epistemological and didactic points of view.

Final Remarks

The aim of this work was to reflect about the issues that emerged from the Content Analysis based on the productions on *Mathematics Teachers Training* held by the Working Group on Mathematical Education in Higher Education (WG04) of the Brazilian Society of Mathematical Education. We identified six great pillars which the discussed topics circle around: (*i*) the good Mathematics teacher; (*ii*) the initial training of the Mathematics teacher; (*iii*) the supervised internship practice; (*iv*) the training and the concerns to be manifested in the work of an educator of Mathematics teachers; (*v*) the professional development of the teacher; and (*vi*) researches about one's own practice in Higher Education. Table 2 shows the distribution of the analyzed work relating to these pillars.

Table 2 – Distribution of the Analyzed Researches, by Topic

Analyzed Researchs	Themes					
	i	ii	iii	iv	v	vi
Menezes (2000)	X					
Viana (2009)	X	X				
Onuchic e Allevato (2009a)	X	X		X	X	
Onuchic e Allevato (2009b)	X	X		X	X	
Palis (2009)						X
Carvalho (2013)		X	X			
Onuchic e Morais (2013)	X	X				
Onuchic e Huanca (2013)	X	X		X		X

Source: Drafted by the authors.

The data presented in Table 2 highlights that the topics that concentrate most reflections refer to the specific characteristics expected from a good Mathematics teacher (a topic that permeates six works) and the initial training of the Mathematics teacher (also present in six researches). Four of the works that discuss the initial training also emphasize concerns about what characterizes a good Mathematics teacher, what in our opinion is expected, as the main objective of every Mathematics undergraduate program should effectively be forming competent teachers in this science. Reflections about the initial training of the teacher that will teach Mathematics in K-12 Education are present in only one of the analyzed researches, which does not dedicate attention to the desired characteristics for this type of teacher.

Among the five works that align discussions about the initial training of the Mathematics teacher and regarding the specific characteristics of these teachers, only one focuses on these two aspects alone. Two also highlight issues regarding the training and the concerns manifested in the work by an educator of Mathematics teachers and the professional development of the teacher. One includes analyses of the training and the concerns manifested in the work of an educator of Mathematics teachers, as well as the researches about one's own practice by the teacher in Higher Education.

Only one research presents considerations about the supervised internship practice, with analyses about this topic and the initial training of the Mathematics teacher. Thus, it is clear that there is a gap, with need of a bigger number of future researches by members of WG04 regarding supervised practice, as well as studies that articulate the other pertinent topics besides initial training. The data shown in Table 2 similarly highlights the reduced presence of investigations regarding *Research about one's own practice* by Higher Education professors – further, of works that seek to bond this topic to the other pillars highlighted in the present analysis, especially regarding the professional development of the teacher and the concerns manifested by the work of an educator of teachers.

In relation to the topic referring to *a good Mathematics teacher*, the discussions converge to the need of content knowledge to be taught, even though this category of teacher knowledge only is not sufficient. The teachers need to know how to deal with knowledge, with technology and with the social complexity generated by the democratization of teaching and with the fact that society is not clear about the effective function of the school.

Concerning the *initial training of the Mathematics teacher*, all the researches indicate that those who prepare the curricular structure of these programs and those who put it in practice should take into consideration the specificities of the professional that will be graduated. This includes the need of differentiating the training of Mathematics teachers and the Mathematics Bachelor's Degree. Besides this, it is essential to provide the students with the experience of different methodological approaches in the development of the activities proposed.

Ultimately, the analyzed researches indicate that it is essential to make the students aware that their professional training does not end with their graduation.

Concerning the *supervised internship practice*, the only work that discusses the topic highlights notes that indicate the potentiality of the curricular activity to break with the theory-practice dichotomy and establish a dialectic relation between these dimensions.

Regarding the pillar *training and the concerns manifested by educators of Mathematics teachers*, one of the central issues discussed is who should teach the undergraduate programs: teachers who have both a specialized approach including Mathematical content and Mathematical Education. It is indispensable that these teachers establish relations between the specific mathematical content of Higher Education and those subsequently worked by graduates in K-12 Education. Another issue to be considered is that some students reach Higher Education with mathematical difficulties, thus being the responsibility of the professors to dedicate moments so that these can be identified and possibly overcome.

About the *professional development of the teacher,* it is necessary to adopt a perspective through which teachers have an active role in their own education. It is also important to incorporate strategies of education, action and research that are pertinent to the teachers and that are effectively useful for their professional practices.

Lastly, concerning the sixth and last pillar, only two works discuss *Researches about One's Own Practice in Higher Education*. The essence of these reflections is the need of debating and stimulating teachers' research about their own practice. We finish this article emphasizing that it is fundamental that there is incentive for the teacher to see with a new light their pedagogical efforts.

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